

CBSE Class 11 Mathematics
Important Questions
Chapter 5
Complex Numbers and Quadratic Equations

1 Marks Questions

1. Evaluate i^{-39}

$$\begin{aligned}\text{Ans. } i^{-39} &= \frac{1}{i^{39}} = \frac{1}{(i^4)^9 \cdot i^3} \\ &= \frac{1}{1 \times (-i)} \quad \left[\begin{array}{l} \because i^4 = 1 \\ i^3 = -i \end{array} \right] \\ &= \frac{1}{-i} \times \frac{i}{i} \\ &= \frac{i}{-i^2} = \frac{i}{-(-1)} = i \quad \left[\because i^2 = -1 \right]\end{aligned}$$

2. Solved the quadratic equation $x^2 + x + \frac{1}{\sqrt{2}} = 0$

$$\begin{aligned}\text{Ans. } \frac{x^2}{1} + \frac{x}{1} + \frac{1}{\sqrt{2}} &= 0 \\ \frac{\sqrt{2}x^2 + \sqrt{2}x + 1}{\sqrt{2}} &= \frac{0}{1} \\ \sqrt{2}x^2 + \sqrt{2}x + 1 &= 0\end{aligned}$$

$$\begin{aligned}
 x &= \frac{-b \pm \sqrt{D}}{2a} \\
 &= \frac{-\sqrt{2} \pm \sqrt{2-4\sqrt{2}}}{2 \times \sqrt{2}} \\
 &= \frac{-\sqrt{2} \pm \sqrt{2}\sqrt{1-2\sqrt{2}}}{2\sqrt{2}} \\
 &= \frac{-1 \pm \sqrt{2\sqrt{2}-1} i}{2}
 \end{aligned}$$

3. If $\left(\frac{1+i}{1-i}\right)^m = 1$, then find the least positive integral value of m.

Ans. $\left(\frac{1+i}{1-i}\right)^m = 1$

$$\left(\frac{1+i}{1-i} \times \frac{1+i}{1+i}\right)^m = 1$$

$$\left(\frac{1+i^2+2i}{1-i^2}\right)^m = 1$$

$$\left(\frac{1-1+2i}{2}\right)^m = 1 \quad \left[\because i^2 = -1 \right]$$

$$i^m = 1$$

$$m=4$$

4. Evaluate $(1+i)^4$

Ans. $(1+i)^4 = \left[(1+i)^2 \right]^2$

$$= (1+i^2+2i)^2$$

$$= (1-1+2i)^2$$

$$= (2i)^2 = 4i^2$$

$$= 4(-1) = -4$$

5. Find the modulus of $\frac{1+i}{1-i} - \frac{1-i}{1+i}$

Ans. Let $z = \frac{1+i}{1-i} - \frac{1-i}{1+i}$

$$= \frac{(1+i)^2 - (1-i)^2}{(1-i)(1+i)}$$

$$= \frac{4i}{2}$$

$$= 2i$$

$$z = 0 + 2i$$

$$|z| = \sqrt{(0)^2 + (2)^2}$$

$$= 2$$

6. Express in the form of a + ib. $(1+3i)^{-1}$

Ans. $(1+3i)^{-1} = \frac{1}{1+3i} \times \frac{1-3i}{1-3i}$

$$\begin{aligned}
&= \frac{1-3i}{(1)^2 - (3i)^2} \\
&= \frac{1-3i}{1-9i^2} \\
&= \frac{1-3i}{1+9} \quad [i^2 = -1] \\
&= \frac{1-3i}{10} \\
&= \frac{1}{10} - \frac{3i}{10}
\end{aligned}$$

7. Explain the fallacy in $-1 = i \cdot i = \sqrt{-1} \cdot \sqrt{-1} = \sqrt{(-1)(-1)} = \sqrt{1} = 1$

Ans. $1 = \sqrt{1} = \sqrt{(-1)(-1)}$ is okay but

$\sqrt{(-1)(-1)} = \sqrt{-1} \sqrt{-1}$ is wrong.

8. Find the conjugate of $\frac{1}{2-3i}$

Ans. Let $z = \frac{1}{2-3i}$

$$z = \frac{1}{2-3i} \times \frac{2+3i}{2+3i}$$

$$= \frac{2+3i}{(2)^2 - (3i)^2}$$

$$= \frac{2+3i}{4+9}$$

$$= \frac{2+3i}{13}$$

$$z = \frac{2}{13} + \frac{3}{13}i$$

$$\bar{z} = \frac{2}{13} - \frac{3}{13}$$

9. Find the conjugate of $-3i - 5$.

Ans. Let $z = 3i - 5$

$$\bar{z} = 3i - 5$$

10. Let $z_1 = 2 - i$, $z_2 = -2 + i$ Find $\operatorname{Re} \left(\frac{z_1 z_2}{z_1} \right)$

Ans. $z_1 z_2 = (2 - i)(-2 + i)$

$$= -4 + 2i + 2i - i^2$$

$$= -4 + 4i + 1$$

$$= 4i - 3$$

$$\bar{z}_1 = 2 + i$$

$$\frac{z_1 z_2}{\bar{z}_1} = \frac{4i - 3}{2 + i} \times \frac{2 - i}{2 - i}$$

$$= \frac{8i - 6 - 4i^2 + 3i}{4 - i^2}$$

$$= \frac{11i - 2}{5}$$

$$\frac{z_1 z_2}{z_1} = \frac{11}{5}i - \frac{2}{5}$$

$$\operatorname{Re}\left(\frac{z_1 z_2}{\bar{z}_1}\right) = -\frac{2}{5}$$

11. Express in the form of $a + ib$ $(3i-7) + (7-4i) - (6+3i) + i^{23}$

Ans. Let

$$Z = \cancel{3i} - \cancel{7} + \cancel{7} - 4i - 6 - \cancel{3i} + (i^4)^5 i^3$$

$$= -4i - 6 - i \quad \left[\begin{array}{l} \because i^4 = 1 \\ i^3 = -i \end{array} \right]$$

$$= -5i - 6$$

$$= -6 + (-5i)$$

12. Find the conjugate of $\sqrt{-3} + 4i^2$

Ans. Let $z = \sqrt{-3} + 4i^2$

$$= \sqrt{3} i - 4$$

$$\bar{z} = -\sqrt{3} i - 4$$

13. Solve for x and y , $3x + (2x-y)i = 6 - 3i$

Ans. $3x = 6$

$$x = 2$$

$$2x - y = -3$$

$$2 \times 2 - y = -3$$

$$-y = -3 - 4$$

$$y = 7$$



14. Find the value of $1+i^2+i^4+i^6+i^8+\dots+i^{20}$

Ans. $1+i^2+(i^2)^2+(i^2)^3+(i^2)^4+\dots+(i^2)^{10}=1$ [$\because i^2 = -1$]

15. Multiply $3-2i$ by its conjugate.

Ans. Let $z = 3 - 2i$

$$\bar{z} = 3 + 2i$$

$$z \bar{z} = (3 - 2i)(3 + 2i)$$

$$= 9 + \cancel{6i} - \cancel{6i} - 4i^2$$

$$= 9 - 4(-1)$$

$$= 13$$

16. Find the multiplicative inverse $4 - 3i$.

Ans. Let $z = 4 - 3i$

$$\bar{z} = 4 + 3i$$

$$|z| = \sqrt{16 + 9} = 5$$

$$z^{-1} = \frac{\bar{z}}{|z|^2}$$

$$= \frac{4 + 3i}{25}$$

$$= \frac{4}{25} + \frac{3}{25}i$$

17. Express in term of $a + ib$

$$\frac{(3+i\sqrt{5})(3-i\sqrt{5})}{(\sqrt{3}+\sqrt{2}i)-(\sqrt{3}-i\sqrt{2})}$$

$$\text{Ans.} = \frac{(3)^2 - (i\sqrt{5})^2}{\sqrt{3} + \sqrt{2}i - \sqrt{3} + i\sqrt{2}}$$

$$= \frac{9+5}{2\sqrt{2}i} = \frac{14}{2\sqrt{2}i}$$

$$= \frac{7}{\sqrt{2}i} \times \frac{\sqrt{2}i}{\sqrt{2}i} = \frac{7\sqrt{2}i}{-2}$$

18. Evaluate $i^n + i^{n+1} + i^{n+2} + i^{n+3}$

$$\text{Ans.} = i^n + i^n i^1 + i^n i^2 + i^n i^3$$

$$= i^n + i^n i - i^n + i^n (-i) \quad \begin{cases} i^3 = -i \\ i^2 = -1 \end{cases}$$

$$= 0$$

19. If 1, w, w² are three cube root of unity, show that (1 - w + w²) (1 + w - w²) = 4

$$\text{Ans.} (1 - w + w^2) (1 + w - w^2)$$

$$(1 + w^2 - w) (1 + w - w^2)$$

$$(-w - w) (-w^2 - w^2) \quad \begin{cases} \because 1 + w = -w^2 \\ 1 + w^2 = -w \end{cases}$$

$$(-2w) (-2w^2)$$

$$4w^3 \quad [w^3 = 1]$$

$$4 \times 1$$

$$= 4$$

20. Find that sum product of the complex number $-\sqrt{3} + \sqrt{2}i$ and $2\sqrt{3} - i$

Ans. $z_1 + z_2 = -\sqrt{3} + \sqrt{2}i + 2\sqrt{3} - i$

$$= \sqrt{3} + (\sqrt{2} - 1)i$$

$$z_1 z_2 = (-\sqrt{3} + \sqrt{2}i)(2\sqrt{3} - i)$$

$$= -6 + \sqrt{3}i + 2\sqrt{6}i - \sqrt{2}i^2$$

$$= -6 + \sqrt{3}i + 2\sqrt{6}i + \sqrt{2}$$

$$= (-6 + \sqrt{2}) + (\sqrt{3} + 2\sqrt{6})i$$

21. Write the real and imaginary part $1 - 2i^2$

Ans. Let $z = 1 - 2i^2$

$$= 1 - 2(-1)$$

$$= 1 + 2$$

$$= 3$$

$$= 3 + 0.i$$

$$\text{Re}(z) = 3, \text{Im}(z) = 0$$

22. If two complex number z_1, z_2 are such that $|z_1| = |z_2|$, is it then necessary that $z_1 = z_2$

Ans. Let $z_1 = a + ib$

$$|z_1| = \sqrt{a^2 + b^2}$$

$$z_2 = b + ia$$

$$|z_2| = \sqrt{b^2 + a^2}$$

Hence $|z_1| = |z_2|$ but $z_1 \neq z_2$

23. Find the conjugate and modulus of $\overline{9-i} + \overline{6+i^3} - \overline{9+i^2}$

Ans. Let $z = \overline{9-i} + \overline{6+i^3} - \overline{9+i^2}$

$$= 9+i+6+i-0$$

$$= 5+2i$$

$$\bar{z} = 5-2i$$

$$|z| = \sqrt{(5)^2 + (-2)^2}$$

$$= \sqrt{25+4}$$

$$= \sqrt{29}$$

24. Find the number of non zero integral solution of the equation $|1-i|^x = 2^x$

Ans. $|1-i|^x = 2^x$

$$\left(\sqrt{(1)^2 + (-1)^2}\right)^x = 2^x$$

$$(\sqrt{2})^x = 2^x$$

$$(2)^{\frac{1}{2}x} = 2^x$$

$$\frac{1}{2}x = x$$

$$\frac{1}{2} = 1$$

$$1 = 2$$

Which is false no value of x satisfies.

25. If $(a + ib)(c + id)(e + if)(g + ih) = A + iB$ then show that

$$(a^2 + b^2)(c^2 + d^2)(e^2 + f^2)(g^2 + h^2) = A^2 + B^2$$

$$\text{Ans. } (a + ib)(c + id)(e + if)(g + ih) = A + iB$$

$$\Rightarrow |(a + ib)(c + id)(e + if)(g + ih)| = |A + iB|$$

$$|a + ib||c + id||e + if||g + ih| = |A + iB|$$

$$(\sqrt{a^2 + b^2})(\sqrt{c^2 + d^2})(\sqrt{e^2 + f^2})(\sqrt{g^2 + h^2}) = \sqrt{A^2 + B^2}$$

sq. both side

$$(a^2 + b^2)(c^2 + d^2)(e^2 + f^2)(g^2 + h^2) = A^2 + B^2$$

CBSE Class 12 Mathematics
Important Questions
Chapter 5
Complex Numbers and Quadratic Equations

4 Marks Questions

1. If $x + iy = \frac{a+ib}{a-ib}$ Prove that $x^2 + y^2 = 1$

Ans. $x + iy = \frac{a+ib}{a-ib}$ (i) (Given)

taking conjugate both side

$$x - iy = \frac{a-ib}{a+ib} \quad (\text{ii})$$

(i) $\times$ (ii)

$$(x+iy)(x-iy) = \left(\frac{a+ib}{a-ib}\right) \times \left(\frac{a-ib}{a+ib}\right)$$

$$(x)^2 - (iy)^2 = 1$$

$$x^2 + y^2 = 1$$

$$[i^2 = -1]$$

2. Find real θ such that $\frac{3+2i \sin \theta}{1-2i \sin \theta}$ is purely real.

Ans. $\frac{3+2i \sin \theta}{1-2i \sin \theta} = \frac{3+2i \sin \theta}{1-2i \sin \theta} \times \frac{1+2i \sin \theta}{1+2i \sin \theta}$

$$= \frac{3+6i \sin \theta + 2i \sin \theta - 4 \sin^2 \theta}{1+4 \sin^2 \theta}$$

$$= \frac{3-4 \sin^2 \theta}{1+4 \sin^2 \theta} + \frac{8i \sin \theta}{1+4 \sin^2 \theta}$$

For purely real

$$\text{Im}(z) = 0$$

$$\frac{8 \sin \theta}{1+4 \sin^2 \theta} = 0$$

$$\sin \theta = 0$$

$$\theta = n\pi$$

3. Find the modulus of $\frac{(1+i)(2+i)}{3+i}$

$$\text{Ans. } \left| \frac{(1+i)(2+i)}{3+i} \right| = \frac{|(1+i)| |2+i|}{|3+i|}$$

$$= \frac{(\sqrt{1^2+1^2})(\sqrt{4+1})}{\sqrt{(3)^2+(1)^2}}$$

$$= \frac{(\sqrt{2})(\sqrt{5})}{\sqrt{10}}$$

$$= \frac{\sqrt{2} \times \sqrt{5}}{\sqrt{2} \times \sqrt{5}}$$

$$= 1$$

4. If $|a+ib|=1$, then Show that $\frac{1+b+ai}{1+b-ai} = b+ai$

$$\text{Ans. } |a+ib|=1$$

$$\sqrt{a^2 + b^2} = 1$$

$$a^2 + b^2 = 1$$

$$\frac{1+b+ai}{1+b-ai} = \frac{(1+b)+ai}{(1+b)-ai} \times \frac{(1+b)+ai}{(1+b)+ai}$$

$$= \frac{(1+b)^2 + (ai)^2 + 2(1+b)(ai)}{(1+b)^2 - (ai)^2}$$

$$= \frac{1+b^2+2b-a^2+2ai+2abi}{1+b^2+2a-a^2}$$

$$= \frac{(a^2+b^2)+b^2+2b-a^2+2ai+2abi}{(a^2+b^2)+b^2+2b-a^2}$$

$$= \frac{2b^2+2b+2ai+2abi}{2b^2+2b}$$

$$= \frac{b^2+b+ai+abi}{b^2+b}$$

$$= \frac{b(b+1)+ai(b+1)}{b(b+1)}$$

$$= b+ai$$

5. If $x - iy = \sqrt{\frac{a-ib}{c-id}}$ Prove that $(x^2 + y^2)^2 = \frac{a^2 + b^2}{c^2 + d^2}$

Ans. $x - iy = \sqrt{\frac{a-ib}{c-id}}$ (1) (Given)

Taking conjugate both side

$$x+iy = \sqrt{\frac{a+ib}{c+id}} \quad (\text{ii})$$

(i) $\times$ (ii)

$$(x-iy) \times (x+iy) = \sqrt{\frac{a-ib}{c-id}} \times \sqrt{\frac{a+ib}{c+id}}$$

$$(x)^2 - (iy)^2 = \sqrt{\frac{(a)^2 - (ib)^2}{(c)^2 - (id)^2}}$$

$$x^2 + y^2 = \sqrt{\frac{a^2 + b^2}{c^2 + d^2}}$$

squaring both side

$$(x^2 + y^2)^2 = \frac{a^2 + b^2}{c^2 + d^2}$$

6.If $a+ib = \frac{c+i}{c-i}$, where a, b, c are real prove that $a^2+b^2 = 1$ and $\frac{b}{a} = \frac{2c}{c^2-1}$

Ans. $a+ib = \frac{c+i}{c-i}$ (Given) (i)

$$a+ib = \frac{c+i}{c-i} \times \frac{c+i}{c+i}$$

$$a+ib = \frac{c^2 + 2ci + i^2}{c^2 - i^2}$$

$$a+ib = \frac{c^2-1}{c^2+1} + \frac{2c}{c^2+1}i$$

$$a = \frac{c^2-1}{c^2+1}, \quad b = \frac{2c}{c^2+1}$$

$$a^2 + b^2 = \left(\frac{c^2 - 1}{c^2 + 1} \right)^2 + \frac{4c^2}{(c^2 + 1)^2}$$

$$= \frac{(c^2 + 1)^2}{(c^2 + 1)^2}$$

$$a^2 + b^2 = 1$$

$$\frac{b}{a} = \frac{2c}{c^2 - 1}$$

$$\frac{b}{a} = \frac{\frac{2c}{c^2 + 1}}{\frac{c^2 - 1}{c^2 + 1}}$$

7. If $z_1 = 2 - i$ and $z_2 = 1 + i$ Find $\left| \frac{z_1 + z_2 + 1}{z_1 - z_2 + i} \right|$

Ans. $z_1 + z_2 + 1 = 2 - i + 1 + i + 1 = 4$

$$z_1 - z_2 + i = 2 - i - 1 - i + i = 1 - i$$

$$\left| \frac{z_1 + z_2 + 1}{z_1 - z_2 + i} \right| = \left| \frac{4}{1 - i} \right|$$

$$= \frac{|4|}{|1 - i|}$$

$$= \frac{4}{\sqrt{1^2 + (-1)^2}}$$

$$= \frac{4}{\sqrt{2}}$$

$$= \frac{4}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{4\sqrt{2}}{2}$$

$$= 2\sqrt{2}$$

8.If $(p + iq)^2 = x + iy$ Prove that $(p^2 + q^2)^2 = x^2 + y^2$

Ans. $(p + iq)^2 = x + iy$ (i)

Taking conjugate both side

$$(p - iq)^2 = x - iy \text{ (ii)}$$

(i) $\times$ (ii)

$$(p + iq)^2 (p - iq)^2 = (x + iy)(x - iy)$$

$$[(p + iq)(p - iq)]^2 = (x)^2 - (iy)^2$$

$$[(p)^2 - (iq)^2]^2 = x^2 - i^2 y^2$$

$$\Rightarrow (p^2 + q^2)^2 = x^2 + y^2$$

9.If $a + ib = \frac{(x+i)^2}{2x^2+1}$ Prove that $a^2 + b^2 = \frac{(x^2+1)^2}{(2x^2+1)^2}$

Ans. $a + ib = \frac{(x+i)^2}{2x^2+1}$ (i) (Given)

Taking conjugate both side

$$a - ib = \frac{(x-i)^2}{2x^2+1} \quad (\text{ii})$$

(i) $\times$ (ii)

$$(a+ib)(a-ib) = \left(\frac{(x+i)^2}{2x^2+1} \right) \times \left(\frac{(x-i)^2}{2x^2+1} \right)$$

$$(a)^2 - (ib)^2 = \frac{(x^2 - i^2)^2}{(2x^2+1)^2}$$

$$a^2 + b^2 = \frac{(x^2+1)^2}{(2x^2+1)^2} \quad \text{proved.}$$

10. If $(x+iy)^3 = u+iv$ then show that $\frac{u}{x} + \frac{v}{y} = 4(x^2 - y^2)$

Ans. $(x+iy)^3 = u+iv$

$$x^3 + (iy)^3 + 3x^2(iy) + 3.x(iy)^2 = u+iv$$

$$x^3 - iy^3 + 3x^2yi - 3xy^2 = u+iv$$

$$x^3 - 3xy^2 + (3x^2y - y^3)i = u+iv$$

$$x(x^2 - 3y^2) + y(3x^2 - y^2)i = u+iv$$

$$x(x^2 - 3y^2) = u, \quad y(3x^2 - y^2) = v$$

$$x^2 - 3y^2 = \frac{u}{x} \quad (\text{i}) \quad \left| \quad 3x^2 - y^2 = \frac{v}{y} \quad (\text{ii}) \right.$$

(i) + (ii)

$$4x^2 - 4y^2 = \frac{u}{x} + \frac{v}{y}$$

$$4(x^2 - y^2) = \frac{u}{x} + \frac{v}{y}$$

11.Solve $\sqrt{3}x^2 - \sqrt{2}x + 3\sqrt{3} = 0$

Ans. $\sqrt{3}x^2 - \sqrt{2}x + 3\sqrt{3} = 0$

$$a = \sqrt{3}, b = -\sqrt{2}, c = 3\sqrt{3}$$

$$D = b^2 - 4ac$$

$$= (-\sqrt{2})^2 - 4 \times \sqrt{3} (3\sqrt{3})$$

$$= 2 - 36$$

$$= -34$$

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$$= \frac{-(-\sqrt{2}) \pm \sqrt{-34}}{2 \times \sqrt{3}}$$

$$= \frac{\sqrt{2} \pm \sqrt{34} i}{2\sqrt{3}}$$

12.Find the modulus $i^{25} + (1+3i)^3$

Ans. $i^{25} + (1+3i)^3$

$$= (i^4)^6 \cdot i + 1 + 27i^3 + 3(1)(3i)(1+3i)$$

$$= i + (1 - 27i + 9i + 27i^2)$$

$$= i + 1 - 18i - 27$$

$$= -26 - 17i$$

$$|i^{25} + (1 + 3i)^3| = |-26 - 17i|$$

$$= \sqrt{(-26)^2 + (-17)^2}$$

$$= \sqrt{676 + 289}$$

$$= \sqrt{965}$$

13. If $a + ib = \frac{(x+i)^2}{2x-i}$ prove that $a^2 + b^2 = \frac{(x^2+1)^2}{4x^2+1}$

Ans. $a + ib = \frac{(x+i)^2}{2x-i}$ (i) (Given)

$a - ib = \frac{(x-i)^2}{2x+i}$ (ii) [taking conjugate both side]

(i) $\times$ (ii)

$$(a + ib)(a - ib) = \frac{(x+i)^2}{(2x-i)} \times \frac{(x-i)^2}{(2x+i)}$$

$$a^2 + b^2 = \frac{(x^2+1)^2}{4x^2+1} \text{ proved.}$$

14. Evaluate $\left[i^{18} + \left(\frac{1}{i} \right)^{25} \right]^3$

Ans. $\left[i^{18} + \left(\frac{1}{i} \right)^{25} \right]^3$

$$\left[(i^4)^4 \cdot i^2 + \frac{1}{i^{25}} \right]^3$$

$$\left[i^2 + \frac{1}{(i^4)^6 \cdot i} \right]^3$$

$$\left[-1 + \frac{1}{i} \right]^3$$

$$\left[-1 + \frac{i^3}{i^4} \right]^3$$

$$\begin{aligned} [-1-i]^3 &= -(1+i)^3 \\ &= -[1^3 + i^3 + 3 \cdot 1 \cdot i(1+i)] \end{aligned}$$

$$= -[1 - i + 3i + 3i^2]$$

$$= -[1 - i + 3i - 3]$$

$$= -[-2 + 2i] = 2 - 2i$$

15. Find that modulus and argument $\frac{1+i}{1-i}$

Ans. $\frac{1+i}{1-i} = \frac{1+i}{1-i} \times \frac{1+i}{1+i}$

$$= \frac{(1+i)^2}{1^2 - i^2}$$

$$= \frac{1+i^2+2i}{1+1}$$

$$= \frac{2i}{2}$$

$$= i$$

$$z = 0 + i$$

$$r = |z| = \sqrt{(0)^2 + (1)^2} = 1$$

Let α be the acute $\angle$ s

$$\tan \alpha = \left| \frac{1}{0} \right|$$

$$\alpha = \pi/2$$

$$\arg(z) = \pi/2$$

$$r = 1$$

16. For what real value of x and y are numbers equal $(1+i)y^2 + (6+i)$ and $(2+i)x$

Ans. $(1+i)y^2 + (6+i) = (2+i)x$

$$y^2 + iy^2 + 6 + i = 2x + xi$$

$$(y^2 + 6) + (y^2 + 1)i = 2x + xi$$

$$y^2 + 6 = 2x$$

$$y^2 + 1 = x$$

$$y^2 = x - 1$$

$$x - 1 + 6 = 2x$$

$$5 = x$$

$$y = \pm 2$$

17.If $x + iy = \sqrt{\frac{1+i}{1-i}}$, prove that $x^2 + y^2 = 1$

Ans. $x + iy = \sqrt{\frac{1+i}{1-i}}$ (i) (Given)

taking conjugate both side

$$x - iy = \sqrt{\frac{1-i}{1+i}} \quad (\text{ii})$$

(i) $\times$ (ii)

$$(x + iy)(x - iy) = \sqrt{\frac{1+i}{1-i}} \times \sqrt{\frac{1-i}{1+i}}$$

$$(x)^2 - (iy)^2 = 1$$

$$x^2 + y^2 = 1$$

Proved.

18.Convert in the polar form $\frac{1+7i}{(2-i)^2}$

Ans. $\frac{1+7i}{(2-i)^2} = \frac{1+7i}{4+i^2-4i} = \frac{1+7i}{3-4i}$

$$= \frac{1+7i}{3-4i} \times \frac{3+4i}{3+4i}$$

$$= \frac{3+4i+21i+28i^2}{9+16}$$

$$= \frac{25i - 25}{25} = i - 1$$

$$= -1 + i$$

$$r = |z| = \sqrt{(-1)^2 + 1^2} = \sqrt{2}$$

Let α be the acute $\angle$ s

$$\tan \alpha = \left| \frac{1}{-1} \right|$$

$$\alpha = \pi/4$$

since $\operatorname{Re}(z) < 0$, $\operatorname{Im}(z) > 0$

$$\theta = \pi - \alpha$$

$$= \pi - \frac{\pi}{4} = 3\pi/4$$

$$z = r(\cos \theta + i \sin \theta)$$

$$= \sqrt{2} \left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4} \right)$$

19. Find the real values of x and y if $(x - iy)(3 + 5i)$ is the conjugate of $-6 - 24i$

Ans.

$$(x - iy)(3 + 5i) = -6 + 24i$$

$$3x + 5xi - 3yi - 5yi^2 = -6 + 24i$$

$$(3x + 5y) + (5x - 3y)i = -6 + 24i$$

$$3x + 5y = -6$$

$$5x - 3y = 24$$

$$x = 3$$

$$y = -3$$

20.If $|z_1| = |z_2| = 1$, prove that $\left| \frac{1}{z_1} + \frac{1}{z_2} \right| = |z_1 + z_2|$

Ans.If $|z_1| = |z_2| = 1$ (Given)

$$\Rightarrow |z_1|^2 = |z_2|^2 = 1$$

$$\Rightarrow z_1 \bar{z}_1 = 1$$

$$\bar{z}_1 = \frac{1}{z_1} \quad (1)$$

$$z_2 \bar{z}_2 = 1$$

$$\bar{z}_2 = \frac{1}{z_2} \quad (2)$$

$$\left[\because z \bar{z} = |z|^2 \right]$$

$$\left| \frac{1}{z_1} + \frac{1}{z_2} \right| = \left| \bar{z}_1 + \bar{z}_2 \right|$$

$$= \left| \overline{z_1 + z_2} \right|$$

$$= |z_1 + z_2|$$

$$\left[\because |\bar{z}| = |z| \text{ proved.} \right]$$

CBSE Class 12 Mathematics
Important Questions
Chapter 5
Complex Numbers and Quadratic Equations

6 Marks Questions

1. If $z = x + iy$ and $w = \frac{1-i^2}{z-i}$ Show that $|w| = 1 \Rightarrow z$ is purely real.

Ans. $w = \frac{1-iz}{z-i}$

$$= \frac{1-i(x+iy)}{x+iy-i}$$

$$= \frac{1-ix-i^2y}{x+i(y-1)}$$

$$= \frac{(1+y)-ix}{x+i(y-1)}$$

$$\therefore |w| = 1$$

$$\Rightarrow \left| \frac{(1+y)-ix}{x+i(y-1)} \right| = 1$$

$$\frac{|(1+y)-ix|}{|x+i(y-1)|} = 1$$

$$\frac{\sqrt{(1+y)^2 + (-x)^2}}{\sqrt{x^2 + (y-1)^2}} = 1$$

$$1+y^2+2y+x^2 = x^2+y^2+1-2y$$

$$4y = 0$$



$$y = 0$$

$$\therefore z = x + i$$

is purely real

2. Convert into polar form $\frac{-16}{1+i\sqrt{3}}$

$$\text{Ans. } \frac{-16}{1+i\sqrt{3}} = \frac{-16}{1+i\sqrt{3}} \times \frac{1-i\sqrt{3}}{1-i\sqrt{3}}$$

$$= \frac{-16(1-i\sqrt{3})}{(1)^2 - (i\sqrt{3})^2}$$

$$= \frac{-16(1-i\sqrt{3})}{1+3}$$

$$= -4(1-i\sqrt{3})$$

$$z = -4 + i4\sqrt{3}$$

$$r = |z| = \sqrt{(-4)^2 + (4\sqrt{3})^2}$$

$$= \sqrt{16 + 48}$$

$$= \sqrt{64}$$

$$= 8$$

Let α be the acute $\angle S$

$$\tan \alpha = \left| \frac{\cancel{4}\sqrt{3}}{\cancel{-4}} \right|$$

$$\tan \alpha = \tan \frac{\pi}{3}$$

$$\alpha = \pi/3$$

Since $\operatorname{Re}(z) < 0$, and $\operatorname{Im}(z) > 0$

$$\theta = \pi - \alpha$$

$$= \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

$$z = 8 \left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3} \right)$$

3. Find two numbers such that their sum is 6 and the product is 14.

Ans. Let x and y be the no.

$$x + y = 6$$

$$xy = 14$$

$$x^2 - 6x + 14 = 0$$

$$D = -20$$

$$x = \frac{-(-6) \pm \sqrt{-20}}{2 \times 1}$$

$$= \frac{6 \pm 2\sqrt{5}i}{2}$$

$$= 3 \pm \sqrt{5}i$$

$$x = 3 + \sqrt{5}i$$

$$y = 6 - (3 + \sqrt{5}i)$$

$$= 3 - \sqrt{5}i$$

when $x = 3 - \sqrt{5} i$

$$y = 6 - (3 - \sqrt{5} i)$$

$$= 3 + \sqrt{5} i$$

4. Convert into polar form $z = \frac{i-1}{\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}}$

Ans. $z = \frac{i-1}{\frac{1}{2} + \frac{\sqrt{3}}{2} i}$

$$= \frac{2(i-1)}{1+\sqrt{3}i} \times \frac{1-\sqrt{3}i}{1-\sqrt{3}i}$$

$$z = \frac{\sqrt{3}-1}{2} + \frac{\sqrt{3}+1}{2} i$$

$$r = |z| = \left(\frac{\sqrt{3}-1}{2} \right)^2 + \left(\frac{\sqrt{3}+1}{2} \right)^2$$

$$r = 2$$

Let α be the acute $\angle$ s

$$\tan \alpha = \left| \frac{\frac{\sqrt{3}+1}{2}}{\frac{\sqrt{3}-1}{2}} \right|$$

$$= \left| \frac{\sqrt{3} \left(1 + \frac{1}{\sqrt{3}} \right)}{\sqrt{3} \left(1 - \frac{1}{\sqrt{3}} \right)} \right|$$

$$= \left| \frac{\tan \frac{\pi}{4} + \tan \frac{\pi}{6}}{1 - \tan \frac{\pi}{4} \tan \frac{\pi}{6}} \right|$$

$$\tan \alpha = \left| \tan \left(\frac{\pi}{4} + \frac{\pi}{6} \right) \right|$$

$$\alpha = \frac{\pi}{4} + \frac{\pi}{6} = \frac{5\pi}{12}$$

$$z = 2 \left(\cos \frac{5\pi}{12} + i \sin \frac{5\pi}{12} \right)$$

5.If α and β are different complex number with $|\beta| = 1$ Then find $\left| \frac{\beta - \alpha}{1 - \overline{\alpha}\beta} \right|$

$$\text{Ans. } \left| \frac{\beta - \alpha}{1 - \overline{\alpha}\beta} \right|^2 = \left(\frac{\beta - \alpha}{1 - \overline{\alpha}\beta} \right) \left(\frac{\overline{\beta - \alpha}}{1 - \overline{\overline{\alpha}\beta}} \right) \quad [\because |z|^2 = z\overline{z}]$$

$$\begin{aligned}
&= \left(\frac{\beta - \alpha}{1 - \overline{\alpha}\beta} \right) \left(\frac{\overline{\beta} - \overline{\alpha}}{1 - \alpha\overline{\beta}} \right) \\
&= \left(\frac{\beta\overline{\beta} - \beta\overline{\alpha} - \alpha\overline{\beta} + \alpha\overline{\alpha}}{1 - \alpha\overline{\beta} - \overline{\alpha}\beta + \alpha\overline{\alpha}\beta\overline{\beta}} \right) \\
&= \left(\frac{|\beta|^2 - \beta\overline{\alpha} - \alpha\overline{\beta} + |\alpha|^2}{1 - \alpha\overline{\beta} - \overline{\alpha}\beta + |\alpha|^2 |\beta|^2} \right) \\
&= \left(\frac{1 - \beta\overline{\alpha} - \alpha\overline{\beta} + |\alpha|^2}{1 - \alpha\overline{\beta} - \overline{\alpha}\beta + |\alpha|^2} \right) \quad [\because |\beta|=1] \\
&= 1
\end{aligned}$$

$$\left| \frac{\beta - \alpha}{1 - \overline{\alpha}\beta} \right| = \sqrt{1}$$

$$\left| \frac{\beta - \alpha}{1 - \overline{\alpha}\beta} \right| = 1$$